

12-2

The Pythagorean Theorem

MAIN IDEA

Find length using the Pythagorean Theorem.

New Vocabulary

leg
hypotenuse
Pythagorean Theorem

Math Online

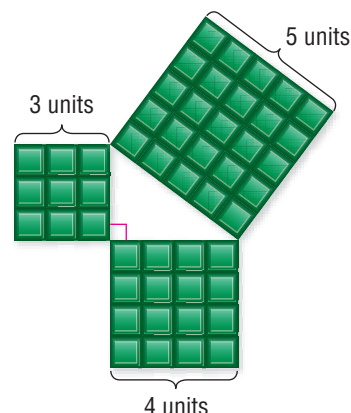
glencoe.com

- Extra Examples
- Personal Tutor
- Self-Check Quiz

MINI Lab

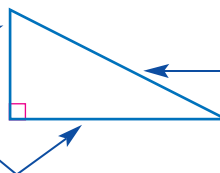
Three squares with sides 3, 4, and 5 units are used to form the right triangle shown.

1. Find the area of each square.
2. How are the squares of the sides related to the areas of the squares?
3. Find the sum of the areas of the two smaller squares. How does the sum compare to the area of the larger square?
4. Use grid paper to cut out three squares with sides 5, 12, and 13 units. Form a right triangle with these squares. Compare the sum of the areas of the two smaller squares with the area of the larger square.



In a right triangle, the sides have special names.

The two sides that form the right angle are the **legs**.



The side opposite the right angle is the **hypotenuse**. It is the longest side of the triangle.

The **Pythagorean Theorem** describes the relationship between the length of the hypotenuse and the lengths of the legs.

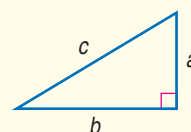
Pythagorean Theorem

Key Concept

Words

In a right triangle, the square of the length of the hypotenuse equals the sum of the squares of the lengths of the legs.

Model



Symbols

$$c^2 = a^2 + b^2$$

When using the Pythagorean Theorem, you will encounter equations that involve square roots. Every positive number has both a positive and a negative square root. By the definition of square roots, if $n^2 = a$, then $n = \pm\sqrt{a}$. The notation $\pm\sqrt{}$ indicates both the positive and negative square root of a number. You can use this relationship to solve equations that involve squares.

Study Tip

Check for Reasonableness
You can eliminate -8.9 as a solution because the length of a side of a triangle cannot be a negative number.

EXAMPLE

Find the Length of the Hypotenuse

- 1 Find the length of the hypotenuse of the triangle.

$$c^2 = a^2 + b^2$$

Pythagorean Theorem

$$c^2 = 8^2 + 4^2$$

Replace a with 8 and b with 4.

$$c^2 = 64 + 16$$

Evaluate 8^2 and 4^2 .

$$c^2 = 80$$

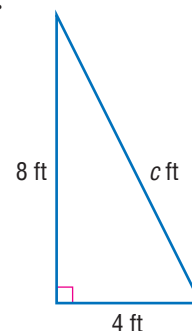
Add.

$$c = \pm\sqrt{80}$$

Definition of square root

$$c \approx \pm 8.9$$

Simplify.



The length of the hypotenuse is about 8.9 feet.

CHECK Your Progress

- a. Find the length of the hypotenuse of a right triangle with legs 5 yards and 7 yards. Round to the nearest tenth.

Real-World EXAMPLE

- 2 **SCUBA DIVING** A scuba diver dove 14 feet below the surface. Then, he swam 16 feet toward a coral formation. How far is the diver from his boat?

The diver's distance from the boat is the hypotenuse of a right triangle. Write and solve an equation for x .

$$c^2 = a^2 + b^2$$

Pythagorean Theorem

$$x^2 = 14^2 + 16^2$$

Replace c with x , a with 14, and b with 16.

$$x^2 = 196 + 256$$

Evaluate 14^2 and 16^2 .

$$x^2 = 452$$

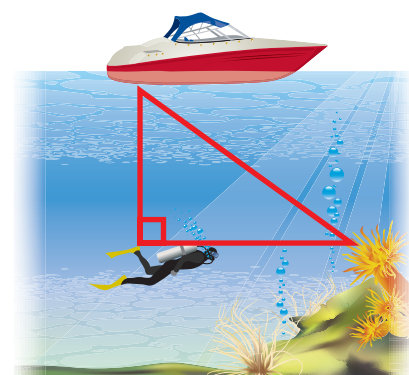
Add.

$$x = \pm\sqrt{452}$$

Definition of square root

$$x \approx \pm 21.3$$

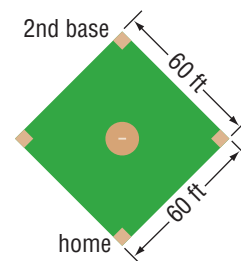
Simplify.



The diver's distance from the boat is about 21.3 feet.

CHECK Your Progress

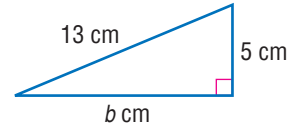
- b. **SOFTBALL** A softball diamond is a square measuring 60 feet on each side. How far does a player on second base throw when she throws from second base to home? Round to the nearest tenth.



You can also use the Pythagorean Theorem to find the measure of a leg if the measure of the other leg and the hypotenuse are known.

EXAMPLE Find the Length of a Leg

- 3** Find the missing measure of the triangle. Round to the nearest tenth if necessary.



The missing measure is the length of a leg.

$$c^2 = a^2 + b^2 \quad \text{Pythagorean Theorem}$$

$$13^2 = 5^2 + b^2 \quad \text{Replace } a \text{ with 5 and } c \text{ with 13.}$$

$$169 = 25 + b^2 \quad \text{Evaluate } 13^2 \text{ and } 5^2.$$

$$-25 = -25 \quad \text{Subtract 25 from each side.}$$

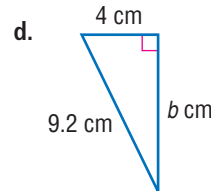
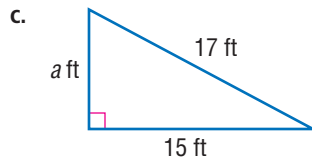
$$144 = b^2 \quad \text{Simplify.}$$

$$\pm\sqrt{144} = b \quad \text{Definition of square root}$$

$$12 = b \quad \text{Simplify.}$$

The length of the leg is 12 centimeters.

CHECK Your Progress



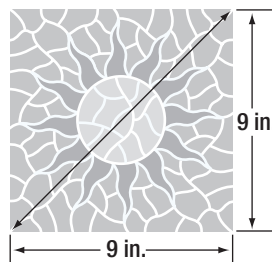
e. $b = 7$ in., $c = 25$ in.

Test-Taking Tip

Formulas Some formulas will be given to you during the test. It is a good idea to familiarize yourself with the formulas before the test.

TEST EXAMPLE

- 4** Mr. Thomson created a mosaic tile in the shape of a square to place in his kitchen.



Which is closest to the length of the diagonal of the tile?

- A 10 in. C 15 in.
B 13 in. D 17 in.

Read the Item

You need to use the Pythagorean Theorem to find the length of the diagonal.

Solve the Item

$$c^2 = a^2 + b^2$$

Pythagorean Theorem

$$c^2 = 9^2 + 9^2$$

Replace a with 9 and b with 9.

$$c^2 = 81 + 81$$

Evaluate 9^2 and 9^2 .

$$c^2 = 162$$

Add.

$$c = \pm\sqrt{162}$$

Definition of square root

$$c \approx \pm 12.7$$

Simplify.

The length is about 12.7 inches.

The answer choice closest to 12.7 inches is 13 inches. So, the answer is B.

CHECK Your Progress

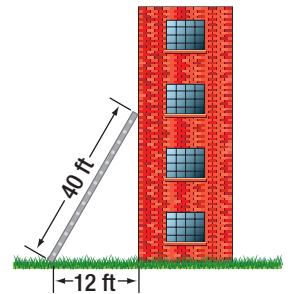
- f. A painter leans a ladder against the side of a building. How far from the bottom of the building is the top of the ladder?

F 38.2 ft

H 21.8 ft

G 28.0 ft

J 20.0 ft

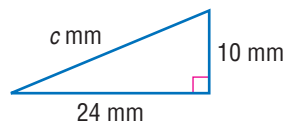


CHECK Your Understanding

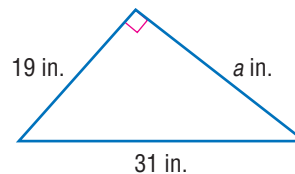
Examples 1, 3
(pp. 641–642)

Find the missing measure of each triangle. Round to the nearest tenth if necessary.

1.



2.

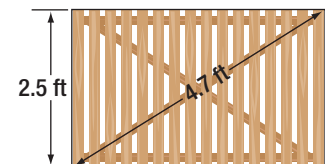


3. $b = 21$ cm, $c = 28$ cm

4. $a = 11$ yd, $b = 12$ yd

Example 2
(p. 641)

5. **ARCHITECTURE** What is the width of the fence gate shown at the right? Round to the nearest tenth.



Example 4
(pp. 642–643)

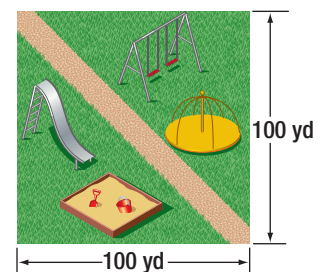
6. **MULTIPLE CHOICE** A company designed a public play area in the shape of a square. The play area will include a pathway, as shown. Which is closest to the length of the pathway?

A 100 yd

C 140 yd

B 125 yd

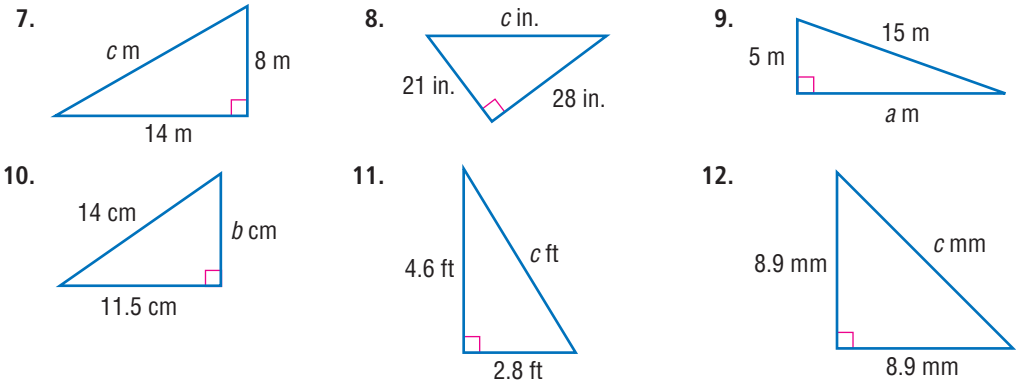
D 175 yd



Practice and Problem Solving

HOMEWORK HELP	
For Exercises	See Examples
7–8, 11–12, 15–16	1
17–20	2
9–10, 13–14	3
26–27	4

Find the missing measure of each triangle. Round to the nearest tenth if necessary.



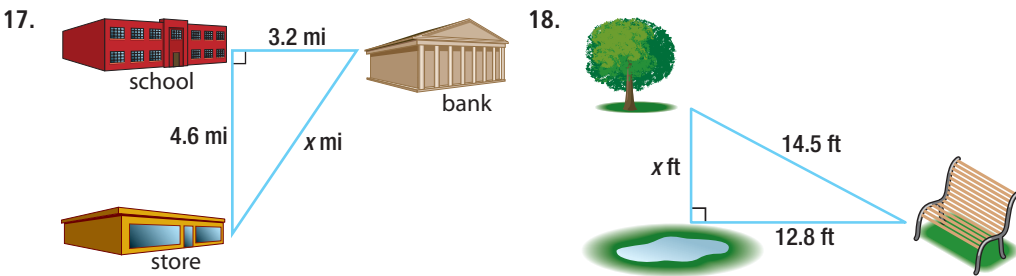
13. $a = 2.4$ yd, $c = 3.7$ yd

14. $b = 8.5$ m, $c = 10.4$ m

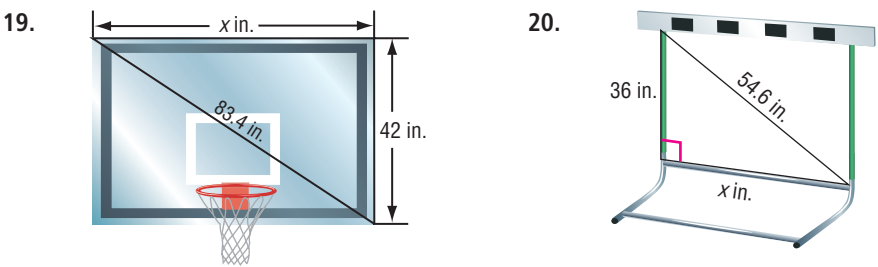
15. $a = 7$ in., $b = 24$ in.

16. $a = 13.5$ mm, $b = 18$ mm

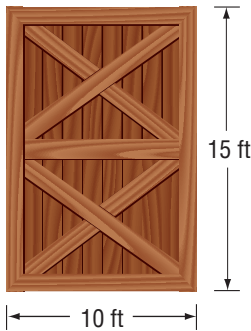
MEASUREMENT For Exercises 17 and 18, find each distance to the nearest tenth.



SPORTS For Exercises 19 and 20, find the length or width of each piece of sports equipment. Round to the nearest tenth.



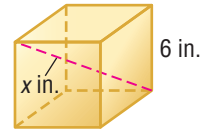
21. **MEASUREMENT** A barn door is 10 feet wide and 15 feet tall. A square plank 16 feet on each side must be taken through the doorway. Can the plank fit through the doorway? Justify your answer.
22. **MEASUREMENT** On a weekend trip around California, Sydney left her home in Modesto and drove 75 miles east to Yosemite National Park, then 70 miles south to Fresno, and finally 110 miles west to Monterey Bay. About how far is she from her starting point? Justify your answer with a drawing.



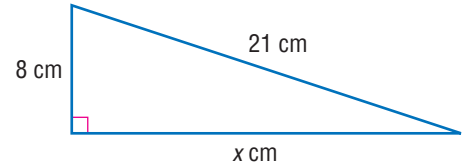
EXTRA PRACTICE
See pages 702, 715.

H.O.T. Problems

23. **CHALLENGE** What is the length of the diagonal shown in the cube at the right?



24. **FIND THE ERROR** Marcus and Aisha are writing an equation to find the missing measure of the triangle at the right. Who is correct? Explain.



Marcus

$$21^2 = 8^2 + x^2$$

$$x^2 = 21^2 + 8^2$$

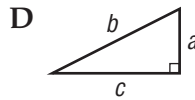
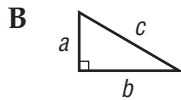
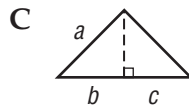
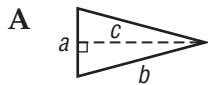


Aisha

25. **WRITING IN MATH** Write a problem about a real-world situation in which you would use the Pythagorean Theorem.

TEST PRACTICE

26. Which triangle has sides a , b , and c so that the relationship $a^2 + b^2 = c^2$ is true?



27. An isosceles right triangle has legs that are each 8 inches long. About how long is the hypotenuse?

- F 12.8 inches
G 11.3 inches
H 8 inches
J 4 inches

Spiral Review

28. **ESTIMATION** Which is closer to $\sqrt{55}$: 7 or 8? (Lesson 12-1)
29. **MEASUREMENT** A cylinder-shaped popcorn tin has a height of 1.5 feet and a diameter of 10 inches. Find the volume to the nearest cubic inch. (Lesson 11-10)

Write each percent as a decimal. (Lesson 4-7)

30. 45%

31. 8%

32. 124%

33. 265%

GET READY for the Next Lesson

34. **PREREQUISITE SKILL** The average person takes about 15 breaths per minute. At this rate, how many breaths does the average person take in one week? Use the *solve a simpler problem* strategy. (Lesson 11-5)